

# NEUTRINO BI-LARGE MIXINGS AND FAMILY <sup>a</sup>

Masako BANDO<sup>b</sup> and Midori OBARA<sup>c</sup>

<sup>1</sup> *Aichi University, Miyoshi Aichi 470-0296, Japan*

<sup>2</sup> *Institute of Humanities and Sciences,  
Ochanomizu University, Tokyo 112-8610, Japan*

After a brief review of quark-lepton relations in grand unified theories (GUT), we show that the Pati-Salam relation with only one type of Higgs field configuration with "four zero symmetric texture" can reproduce two large neutrino mixings as well as observed mass differences. This is quite in contrast to the case of  $SU(5)$  where bi-large mixings essentially come from the charged lepton sector with non-symmetric charged lepton mass matrix.

## 1. Neutrino Masses and GUT

Recent results from KamLAND <sup>1</sup> together with the neutrino experiments by Super-Kamiokande <sup>2,3</sup> and SNO <sup>4</sup> have confirmed neutrino oscillations with two large mixing angles <sup>5,6,7,8</sup> with the mass squared differences are

$$0.29 \leq \tan^2 \theta_{12} \leq 0.86, \quad 5.1 \times 10^{-5} \leq \Delta m_{12}^2 \leq 9.7 \times 10^{-3} \text{ eV}^2, \quad (1.1)$$

$$0.83 < \sin^2 2\theta_{23}, \quad 1.4 \times 10^{-3} \leq \Delta m_{23}^2 \leq 6.0 \times 10^{-3} \text{ eV}^2, \quad (1.2)$$

As we can express the neutrino mixings in terms of MNS matrix <sup>9</sup>, which are further divided into two terms,  $U_l$  and  $U_\nu$ , the unitary matrices which diagonalize the  $3 \times 3$  charged lepton and neutrino mass matrices,  $M_l$  and  $M_\nu$ ;

$$U_{MNS} = U_l U_\nu^\dagger, \quad (1.3)$$

$$U_l^\dagger M_l V_l = \text{diag}(m_e, m_\mu, m_\tau), \quad U_\nu^\dagger M_\nu U_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}), \quad (1.4)$$

in analogous to CKM matrix derived from quark mass matrices,  $M_d$  and  $M_u$ ;

$$U_{CKM} = U_u U_d^\dagger, \quad (1.5)$$

$$U_u^\dagger M_u V_u = \text{diag}(m_u, m_c, m_t), \quad U_d^\dagger M_d V_d = \text{diag}(m_d, m_s, m_b), \quad (1.6)$$

where  $U$  and  $V$  are unitary matrix acting on left- and right-handed fermions, respectively and  $\text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$  are mass eigenvalues of fermions. The

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<sup>b</sup>E-mail address: bando@aichi-u.ac.jp

<sup>c</sup>E-mail address: midori@hep.phys.ocha.ac.jp

observed tiny neutrino masses are most naturally explained if tree mass terms are forbidden by some symmetry and they come only from the higher dimensional operator (the so-called see-saw mechanism), which we adopt in this paper. Then the neutrino mass matrix  $M_\nu$  is derived from huge right-handed Majorana masses ( $M_R$ ) and the Dirac masses ( $M_{\nu_D}$ ) of EW scale;

$$M_\nu = M_{\nu_D}^T M_R^{-1} M_{\nu_D}. \quad (1.7)$$

In the SUSY standard model, the following Yukawa terms, which has family indices  $3 \times 3$ ,  $Y_{i,j}$ , connect  $SU(2)_L$  **2**,  $Q$  and  $L$  to **1** fermions,  $u_R, d_R, e_R, \nu_R$ ;

$$W_Y = Q_L(Y_u u_R H_u + Y_d d_R H_d) + L(Y_l e_R H_u + Y_{\nu_D} \nu_R H_d) + Y_R \nu_R \nu_R H_R. \quad (1.8)$$

The  $SU(2)$  doublet up- and down-type Higgs fields,  $H_u, H_d$  with VEVs,  $v_u, v_d$  give mass matrices  $\rightarrow M_u = Y_u v_u, M_d = Y_d v_d, M_{\nu_D} = Y_{\nu_D} v_u, M_l = Y_l v_d$  after the standard symmetry is broken down to  $SU(3) \times U(1)_{em}$ . The Majorana mass term (the last term of Eq. (1.8) including  $SU(2)$  singlet Higgs field  $H_R$  gives neutrino right-handed mass term,  $M_R = Y_R v_R$  where the VEV  $v_R$  is expected to be much larger than EW scale.

## 2. Hierarchy Problem

Before going to the discussion of neutrino masses, we make comments on the hierarchy problems whose origin may indicate something to the family structure. If one wants to construct unified theory, it is governed by the scale  $M_P$  which is far higher than electroweak scale. We know that the GUT scale,  $M_G$  is near below the Planck scale,  $M_P$  which is usually taken as the reduced Planck mass,  $1/\sqrt{8\pi G_N}$ . This huge discrepancy between two scales,  $M_P$  and  $M_W$  is called "strong hierarchy problem". The introduction of supersymmetry (SUSY) provides a good solution for solving strong hierarchy. The GUT itself needs several energy scales appearing in the steps of GUT breakings into standard symmetry  $G_1 \rightarrow G_2 \cdots G_M$ , with each of scales actually expressed as  $\lambda^n M_P$ . There we use the typical hierarchical parameter  $\lambda \sim 0.2$  (the Cabibbo mixing angle). Especially recent neutrino small masses indicates some intermediate scales of order  $10^{11-13}$  GeV, which is roughly equal to  $\lambda^8 M_P$ . Thus we here recognize "mild hierarchy" appearing as  $\lambda^n M_P$  in terms of the typical hierarchical parameter  $\lambda \sim 0.2$  (the Cabibbo mixing angle). The intermediate scale  $M_R$  is of order  $10^{11-13}$  GeV corresponds to  $\lambda^8 M_P$ . On the other hand, the hierarchical fermion masses are also controlled by the same  $\lambda$ ; the electron mass, the smallest Dirac fermion mass is almost  $\lambda^8 M_t$ , in terms of the top quark mass,  $m_t = 170$  GeV. It is interesting that both in high and low energy regions (at  $M_P$  and  $M_W$  scales) common the power law,  $\lambda^n$  mild hierarchy structure,

which may give a hint of the origin of family. Indeed the simplest example to explain this is to introduce the anomalous  $U(1)_X$  family quantum number: the power structure comes from the Froggatt-Nielsen mechanism according to which Yukawa couplings come from higher dimensional operators  $\lambda^n = (\frac{\theta}{M_P})^n$  with  $n$  determined to compensate the  $U(1)_X$  symmetry by Froggatt-Nielsen field  $\theta(X = -1)$ . This simplest example of family symmetry has a characteristic feature; If we assume the strength of all coupling constants of order 1, the Froggatt-Nielsen mechanism produces the power hierarchical coefficients of  $\lambda^n$  where  $n$  is solely determined by the  $U(1)$  charges of relevant fields and so such power structure is always of factorizable form. Within the framework of GUT the  $X$  charge is assigned to the each GUT multiplet. Also this  $U(1)_X$  explains the "mild hierarchies" of symmetry breaking scales by assigning  $X$  charges to Higgs fields. We could also introduce more complicated family symmetries beyond abelian case. In the following we shall examine how such family quantum numbers can be consistent with recent neutrino experimental data.

### 3. GUT and Family symmetry

Now if we assume some GUT and that the family structure is the same for all the members of a multiplet. Two types of relations are derived between the unitary matrices  $U_{u,d,l,\nu}$  according to the different kinds of symmetries of the system. In the following sections we investigate how the unitary matrices,  $U_d, U_u, U_l$  and  $U_{\nu D}$ , are mutually related with each other if we assume some grand unified gauge theory or some family symmetry.

First, within the standard model, there is no relation between them since the up and down fermions couple to different Higgs fields. If the hierarchical mass structure comes from the Froggatt Nielsen mechanism, then the power factor is determined merely from the  $X$  charges of relevant left-handed fermions. Then, since we have already  $SU(2)_L$  in standard model, the  $SU(2)$ -doublets,  $Q$  and  $L$  have common charges,  $q_1, q_2, q_3$  and  $l_1, l_2, l_3$ , respectively, yielding

$$U_{CKM} \simeq U_d \simeq U_u, \quad U_{MNS} \simeq U_l \simeq U_{\nu D} \simeq U_\nu, \quad (3.9)$$

with  $\simeq$  indicating the same power structure. Second, in the Georgi-Glashow  $SU(5)$  symmetry ( $S(3)_c \times SU(2)_L \times U(1)_Y \subset SU(5)$ ), where  $(Q_L, e_R, u_R)$  belong to the same multiplet, **10**, while  $(L, d_R)$ , to **5\***. Note that  $SU(5)$  GUT never provides new relations for  $M_{CKM}$  and  $M_{NMS}$ , although we have a familiar relation,  $M_l \leftrightarrow M_d^T$ , once we fix the representations of Higgs field,  $M_d \leftrightarrow M_l^T$ , where  $\leftrightarrow$  means that they are mutually related by accompanying some CG coefficients according to the representations of coupled Higgs field.

Thirdly, the Pati-Salam symmetry  $SU(4)_{PS}$  combines  $u_L$  with  $\nu_L$  into  $F_L$  and  $d_L$  with  $l_L$  into  $F_R$ , so we have  $U_{CKM} \simeq U_u \simeq U_d \simeq U_{MNS} \simeq U_l \simeq U_\nu$ . Thus we see that the Pati-Salam symmetry with  $X$ -charge power law does predict the same hierarchical mixing matrix both for  $U_{CKM}$  and  $U_{MNS}$ . Thus in order to reproduce neutrino large mixing angles within Pati-Salam symmetry, we should discard simple Froggatt Nielsen mechanism and give up factorizable property of power law structure. The simplest example of such possibilities is to introduce "zero texture" which has been extensively investigated by many authors<sup>11</sup>. Note that even in such case we have the relation of mass matrices,  $M_{\nu_D} \leftrightarrow M_u$ ,  $M_l \leftrightarrow M_d$ . For larger GUT,  $SO(10)$ , the situation is essentially the same, so far as they include the subgroups above mentioned. In  $SO(10)$  all the fermions of a family may form a single multiplet **16**, and so if the Froggatt Nielsen mechanism works, all the components of  $\psi(\mathbf{16})$  do have common  $X$  charge. So if we take  $\psi_i(\mathbf{16})$  to  $i$ -th family, and since Pati-Salam symmetry dictates small neutrino mixing angles. Thus we must introduce twisted family structure by introducing new fermions  $\psi(\mathbf{10})$  and  $\psi_i(\mathbf{16})$  must not be identified to  $i$ -th family (non-parallel family structure), which is most naturally implemented in  $E_6$  GUT<sup>16</sup>. Leaving the discussion of  $SU(5)$  symmetry to Kugo's talk, I here show an example of the Pati-Salam symmetry in the next section.

#### 4. Pati-Salam Symmetry with Symmetric Four Zero Texture

The model we introduce here is the following example<sup>10</sup> of symmetric "four zero" texture which has been extensively investigated by many authors<sup>11</sup>. Under the Pati-Salam symmetry ( we name this "up-road option" because the neutrino large mixing angle is related to up-quark mass matrix  $M_u$  as mentioned in the previous section). We show that the following configuration of the representation of Higgs field for up-quark mass matrix

$$M_u = \begin{pmatrix} 0 & \mathbf{126} & 0 \\ \mathbf{126} & \mathbf{10} & \mathbf{10} \\ 0 & \mathbf{10} & \mathbf{126} \end{pmatrix}, M_R = \begin{pmatrix} 0 & rM_R & 0 \\ rM_R & 0 & 0 \\ 0 & 0 & M_R \end{pmatrix}. \quad (4.10)$$

which may be compared with the Georgi-Jahrschog of down-type mass matrix. Now  $M_{\nu_D}$  is obtained by multiplying Clebsch-Gordan coefficient, 1 or  $-3$ ;

$$M_u \simeq m_t \begin{pmatrix} 0 & a & 0 \\ a & b & c \\ 0 & c & 1 \end{pmatrix} \leftrightarrow M_{\nu_D} = m_t \begin{pmatrix} 0 & -3a & 0 \\ -3a & b & c \\ 0 & c & -3 \end{pmatrix}, \quad \begin{aligned} a &= \frac{\sqrt{m_u m_c}}{m_t}, \\ b &= \frac{m_c}{m_t}, \\ c &= \sqrt{\frac{m_u}{m_t}}. \end{aligned} \quad (4.11)$$

Then  $M_\nu$  are easily calculated. In order to get large mixing angle  $\theta_{23}$ , the of 2-3 element of  $M_\nu$  should be of the same order of magnitude as 3-3 element, namely,

$h \simeq \frac{-ac}{3r} \sim 1 \rightarrow r \sim \sqrt{\frac{m_c^2 m_c}{3m_t^3}} \sim 10^{-7}$ . Such kind of enhancement mechanism is called "seesaw enhancement"<sup>12,13,8</sup> With tiny  $r$ ,  $M_\nu$  is approximately given,

$$M_\nu = \begin{pmatrix} 0 & \frac{9a^2}{r} & 0 \\ \frac{9a^2}{r} & -6ab & -3ac \\ 0 & -\frac{3ac}{r} & 9 \end{pmatrix} \frac{m_t^2}{m_R} \equiv \begin{pmatrix} 0 & \beta & 0 \\ \beta & \alpha & h \\ 0 & h & 1 \end{pmatrix} \frac{9m_t^2}{m_R}, \quad \begin{aligned} h &= -ac/3r, \\ \alpha &= 2hb/c, \\ \beta &= -3ha/c, \end{aligned} \quad (4.12)$$

where  $\sin^2 2\theta_{23}$  can be made large when  $h \sim \mathcal{O}(1)$ . However it is non-trivial to reproduce both the experimental bound  $\sin^2 2\theta_{12}$  and mass ratio  $m_{\nu_{12}}/m_{\nu_{23}}$  and we should not stay in order-of-magnitude calculation. Since all the matrix elements of  $M_\nu$  are now expressed in terms of up-quark masses with the parameter  $r$  or  $h$ , the neutrino masses and mixing angles at GUT scale are obtained straightforwardly<sup>10</sup>. The obtained formula Eq. (4.12) at GUT scale is to be compared with the neutrino experimental data at GUT scale. Here we estimate the RGE evolution of neutrino mass matrix at  $M_Z$  by using the approximate formula obtained by Haba and Okamura<sup>14</sup>. the transformed expression of  $\alpha, \beta$  and  $h$  of Eq. (4.11) from  $M_R$  to  $m_Z$  scale are given as,

$$\beta \rightarrow \frac{1}{(1 - \epsilon_e)(1 - \epsilon_\mu)}\beta, \quad \alpha \rightarrow \frac{1}{(1 - \epsilon_\mu)^2}\alpha, \quad h \rightarrow \frac{1}{1 - \epsilon_\mu}h. \quad (4.13)$$

where the RGE factors  $\epsilon$ 's is estimated as at most 0.1, We compare the corrected values of neutrino masses and mixing angles at  $M_Z$  scale and the experimental data and found that within the error only the small region of  $h$  near around 1 is consistent with the observed mass ratio and two mixing angles, for which case the final expressions of neutrino masses and mixing angles at GUT scale is,

$$\begin{aligned} \tan^2 2\theta_{23} &\simeq \frac{4}{(1 - 2xy)^2}, \quad \tan^2 2\theta_{12} \simeq \frac{72x^2}{(1 - 2xy)^2}, \quad \sin \theta_{13} \simeq \frac{3x}{(3 + 2xy)}, \\ \frac{m_{\nu_2}}{m_{\nu_3}} &\simeq \frac{(2xy - 1)(1 + \sec \theta_{12})}{(3 + 2xy)}, \quad \frac{m_{\nu_1}}{m_{\nu_2}} \simeq \frac{(1 - \sec \theta_{12})}{(1 + \sec \theta_{12})}, \end{aligned}$$

with  $x = \sqrt{\frac{m_c}{m_t}}, y = \sqrt{\frac{m_c}{m_u}}$ . We leave the detailed calculations in our full paper<sup>10</sup>. and here note that our model predicts not only the order of magnitudes but the exact vales of all neutrino masses and mixing angles. This is remarkable and enables us to predict  $U_{e3} \leq 0.1$  irrespectively of CP phase. It is remarked that our neutrino mass matrix has been determined with almost uniquely determined within error bars of up-quark masses, it can make the prediction of leptogenesis calculation once we fix the CP phase, which are now under calculation by Obara, Kaneko Tanimoto and Bando<sup>15</sup>. In conclusion we

have seen that the up-road option can reproduce the present neutrino experimental data very well. However also down-road option may be also worthwhile to be investigated<sup>16</sup>, in which case the Nature may show "twisted family structure". On the contrary in the case of up-road option it requires "parallel family structure".

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